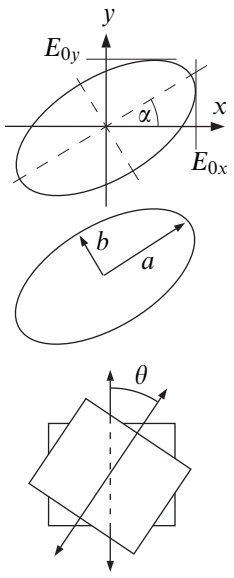
8.6 Polarisation

Elliptical polarisation[a]

Elliptical polarisation	$\boldsymbol{E} = (E_{0x}, E_{0y}\mathrm{e}^{\mathrm{i}\delta})\mathrm{e}^{\mathrm{i}(kz-\omega t)}$ (8.80)	$\boldsymbol{E}$ electric field k wavevector z propagation axis ωt angular frequency × time E_{0x} x amplitude of $\boldsymbol{E}$ E_{0y} y amplitude of $\boldsymbol{E}$ δ relative phase of E_y with respect to E_x α polarisation angle e ellipticity a semi-major axis b semi-minor axis $I(\theta)$ transmitted intensity I_0 incident intensity θ polariser–analyser angle
Polarisation angle[b]	$\tan 2\alpha = \dfrac{2E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$ (8.81)	
Ellipticity[c]	$e = \dfrac{a-b}{a}$ (8.82)	
Malus's law[d]	$I(\theta) = I_0\cos^2\theta$ (8.83)	



[a] See the introduction (page 161) for a discussion of sign and handedness conventions.
[b] Angle between ellipse major axis and x axis. Sometimes the polarisation angle is defined as $\pi/2 - \alpha$.
[c] This is one of several definitions for ellipticity.
[d] Transmission through skewed polarisers for unpolarised incident light.

Jones vectors and matrices

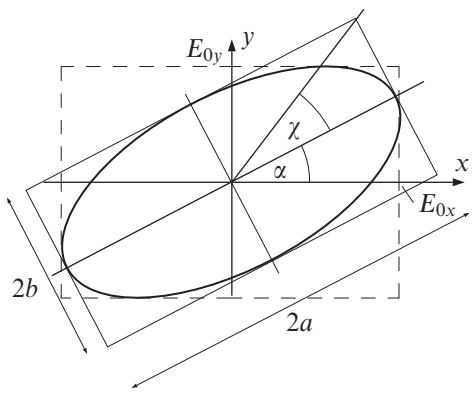
Normalised electric field[a]	$\boldsymbol{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}; \quad \lvert\boldsymbol{E}\rvert = 1$ (8.84)	$\boldsymbol{E}$ electric field E_x x component of $\boldsymbol{E}$ E_y y component of $\boldsymbol{E}$
Example vectors:	$E_x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad E_{45} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $E_\mathrm{r} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -\mathbf{i} \end{pmatrix}$ $\quad E_\mathrm{l} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \mathbf{i} \end{pmatrix}$	E_{45} 45° to x axis E_r right-hand circular E_l left-hand circular
Jones matrix	$\boldsymbol{E}_\mathrm{t} = \mathbf{A}\boldsymbol{E}_\mathrm{i}$ (8.85)	$\boldsymbol{E}_\mathrm{t}$ transmitted vector $\boldsymbol{E}_\mathrm{i}$ incident vector $\mathbf{A}$ Jones matrix

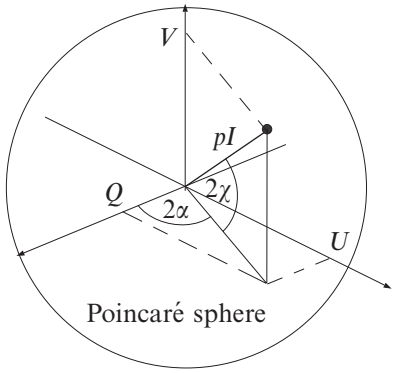
Example matrices:

Linear polariser $\parallel x$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	Linear polariser $\parallel y$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
Linear polariser at 45°	$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	Linear polariser at −45°	$\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$
Right circular polariser	$\frac{1}{2}\begin{pmatrix} 1 & \mathbf{i} \\ -\mathbf{i} & 1 \end{pmatrix}$	Left circular polariser	$\frac{1}{2}\begin{pmatrix} 1 & -\mathbf{i} \\ \mathbf{i} & 1 \end{pmatrix}$
$\lambda/4$ plate (fast $\parallel x$)	$\mathrm{e}^{\mathrm{i}\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & \mathbf{i} \end{pmatrix}$	$\lambda/4$ plate (fast $\perp x$)	$\mathrm{e}^{\mathrm{i}\pi/4}\begin{pmatrix} 1 & 0 \\ 0 & -\mathbf{i} \end{pmatrix}$

[a] Known as the "normalised Jones vector."

Stokes parameters[a]





Electric fields	$E_x = E_{0x}e^{i(kz-\omega t)}$	(8.86)
	$E_y = E_{0y}e^{i(kz-\omega t+\delta)}$	(8.87)
Axial ratio[b]	$\tan\chi = \pm r = \pm\dfrac{b}{a}$	(8.88)
Stokes parameters	$I = \langle E_x^2\rangle + \langle E_y^2\rangle$	(8.89)
	$Q = \langle E_x^2\rangle - \langle E_y^2\rangle$	(8.90)
	$= pI\cos 2\chi\cos 2\alpha$	(8.91)
	$U = 2\langle E_x E_y\rangle\cos\delta$	(8.92)
	$= pI\cos 2\chi\sin 2\alpha$	(8.93)
	$V = 2\langle E_x E_y\rangle\sin\delta$	(8.94)
	$= pI\sin 2\chi$	(8.95)
Degree of polarisation	$p = \dfrac{(Q^2+U^2+V^2)^{1/2}}{I} \le 1$	(8.96)

k	wavevector
ωt	angular frequency × time
δ	relative phase of E_y with respect to E_x
χ	(see diagram)
r	axial ratio
E_x	electric field component $\parallel x$
E_y	electric field component $\parallel y$
E_{0x}	field amplitude in x direction
E_{0y}	field amplitude in y direction
α	polarisation angle
p	degree of polarisation
$\langle\cdot\rangle$	mean over time

	Q/I	U/I	V/I
left circular	0	0	−1
linear $\parallel x$	1	0	0
linear 45° to x	0	1	0
unpolarised	0	0	0

	Q/I	U/I	V/I
right circular	0	0	1
linear $\parallel y$	−1	0	0
linear −45° to x	0	−1	0

[a]Using the convention that right-handed circular polarisation corresponds to a clockwise rotation of the electric field in a given plane when looking towards the source. The propagation direction in the diagram is out of the plane. The parameters I, Q, U, and V are sometimes denoted s_0, s_1, s_2, and s_3, and other nomenclatures exist. There is no generally accepted definition – often the parameters are scaled to be dimensionless, with $s_0 = 1$, or to represent power flux through a plane $\perp$ the beam, i.e., $I = (\langle E_x^2\rangle + \langle E_y^2\rangle)/Z_0$ etc., where Z_0 is the impedance of free space.

[b]The axial ratio is positive for right-handed polarisation and negative for left-handed polarisation using our definitions.